INR 0945c/99

Observability and Probability of Discovery in Future Experiments

S.I. Bityukov[1,2] , N.V. Krasnikov[3]
Institute for Nuclear Research, Moscow, Russia

Abstract

We propose a method to estimate the probability of new physics discovery in future high energy physics experiments. Physics simulation gives both the average numbers $< N_b >$ of background and $< N_s >$ of signal events. We find that the proper definition of the significance is $S_{12} = \sqrt{< N_s > + < N_b >} - \sqrt{< N_b >}$ in comparison with often used significances $S_1 = \dfrac{< N_s >}{\sqrt{< N_b >}}$ and $S_2 = \dfrac{< N_s >}{\sqrt{< N_s > + < N_b >}}$. We propose a method for taking into account systematic uncertainties related to nonexact knowledge of background and signal cross sections. An account of such systematics is very essential in the search for supersymmetry at LHC. We propose a method for estimation of exclusion limits on new physics in future experiments. We also estimate the probability of new physics discovery in future experiments taking into account systematical errors.

[1]Institute for High Energy Physics, Protvino, Moscow region, Russia
[2]E-mails: bityukov@mx.ihep.su, Serguei.Bitioukov@cern.ch
[3]E-mails: krasniko@ms2.inr.ac.ru, Nikolai.Krasnikov@cern.ch

1 Introduction

One of the common goals in the forthcoming experiments is the search for new phenomena. In the forthcoming high energy physics experiments (LHC, TEV22, NLC, ...) the main goal is the search for physics beyond the Standard Model (supersymmetry, Z'-, W'-bosons, ...) and the Higgs boson discovery as a final confirmation of the Standard Model. In estimation of the discovery potential of the future experiments (to be specific in this paper we shall use as an example CMS experiment at LHC [1]) the background cross section is calculated and for the given integrated luminosity L the average number of background events is $< N_b >= \sigma_b \cdot L$. Suppose the existence of a new physics leads to the nonzero signal cross section σ_s with the same signature as for the background cross section that results in the prediction of the additional average number of signal events $< N_s >= \sigma_s \cdot L$ for the integrated luminosity L.

The total average number of the events is $< N_{ev} >=< N_s > + < N_b >= (\sigma_s + \sigma_b) \cdot L$. So, as a result of new physics existence, we expect an excess of the average number of events. In real experiments the probability of the realization of n events is described by Poisson distribution [2]

$$f(n, < n >) = \frac{< n >^n}{n!} e^{-<n>}. \tag{1}$$

Here $< n >$ is the average number of events.

Remember that the Poisson distribution $f(n, < n >)$ gives [3] the probability of finding exactly n events in the given interval of (e.g. space and time) when the events occur independently of one another at an average rate of $< n >$ per the given interval. For the Poisson distribution the variance σ^2 equals to $< n >$. So, to estimate the probability of the new physics discovery we have to compare the Poisson statistics with $< n >=< N_b >$ and $< n >=< N_b > + < N_s >$. Usually, high energy physicists use the following "significances" for testing the possibility to discover new physics in an experiment:

(a) "significance" $S_1 = \dfrac{< N_s >}{\sqrt{< N_b >}}$ [4],

(b) "significance" $S_2 = \dfrac{< N_s >}{\sqrt{< N_s > + < N_b >}}$ [5, 6].

A conventional claim is that for S_1 (S_2) ≥ 5 we shall discover new physics (here, of course, the systematical errors are ignored). For $N_b \gg N_s$ the significances S_1 and S_2 coincide (the search for Higgs boson through the $H \to \gamma\gamma$ signature). For the case when $N_s \sim N_b$, S_1 and S_2 differ. Therefore, a natural question arises: what is the correct definition for the significance S_1, S_2 or anything else ?

It should be noted that there is a crucial difference between "future" experiment and the "real" experiment. In the "real" experiment the total number of events N_{ev} is a given number (already has been measured) and we compare it with $< N_b >$ when we test the validity of the standard physics. So, the number of possible signal events is determined as $N_s = N_{ev} - < N_b >$ and it is compared with the average number of background events $< N_b >$. The fluctuation of the background is $\sigma_{fb} = \sqrt{N_b}$, therefore, we come to the S_1 significance as the measure of the distinction from the standard physics. In the conditions of the "future" experiment when we want to search for new physics, we know only the average number of the background events and the average number of the signal events, so we have to compare the Poisson distributions $P(n, < N_b >)$ and $P(n, < N_b > + < N_s >)$ to determine the probability to find new physics in the future experiment.

In this paper we estimate the probability to discover new physics in future experiments. We show that the proper determination of the significance is $S = \sqrt{< N_s > + < N_b >} - \sqrt{< N_b >}$. We suggest a method which takes into account systematic errors related to nonexact knowledge of the signal and background cross sections. We also propose a method for the estimation of exclusion limits on new physics in future experiments. Some of presented results has been published in our early paper [8].

The organization of the paper is the following. In the next Section we give a method for the determination of the probability to find new physics in the future experiment and calculate the probability to discover new physics for the given $(< N_b >, \ < N_s >)$ numbers of background and signal events under the assumption that there are no systematic errors. In Section 3 we estimate the influence of the systematics related to nonexact knowledge of the signal and background cross sections on the probability to discover new physics in future experiments. In Section 4 we describe a method for the estimation of exclusion limits on new physics in future experiments. In Section 5 we estimate the probability of new physics discovery in future experiments. Section 6 contains concluding remarks.

2 An analysis of statistical fluctuations

Suppose that for some future experiment we know the average number of the background and signal events $< N_b >, \ < N_s >$. As it has been mentioned in the Introduction, the probability of realization of n events in an experiment is given by the Poisson distribution

$$P(n, <n>) = \frac{<n>^n}{n!} e^{-<n>}, \tag{2}$$

where $<n> = <N_b>$ for the case of the absence of new physics and $<n> = <N_b> + <N_s>$ for the case when new physics exists. So, to determine the probability to discover new physics in future experiment, we have to compare the Poisson distributions with $<n> = <N_b>$ (standard physics) and $<n> = <N_b> + <N_s>$ (new physics).

Consider, at first, the case when $<N_b> \gg 1$, $<N_s> \gg 1$. In this case the Poisson distributions approach the Gaussian distributions [1]

$$P_G(n, \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(n-\mu)^2}{2\sigma^2}}, \tag{3}$$

with $\mu = \sigma^2$ and $\mu = <N_b>$ or $\mu = <N_b> + <N_s>$. Here n is a real number. Note that for the Poisson distribution the mean equals to the variance.

The Gaussian distribution describes the probability density to realize n events in the future experiment provided the average number of events $<n>$ is a given number. In Fig.1 we show two Gaussian distributions P_G with $<n> = <N_b> = 53$ and $<n> = <N_b> + <N_s> = 104$ ([6], Table.13, cut 6). As is clear from Fig.1 the common area for these two curves (the first curve shows the "standard physics" events distribution and the second one gives the "new physics" events distribution) is the probability that "new physics" can be described by the "standard physics". In other words, suppose we know for sure that new physics takes place and the probability density of the events realization is described by curve II ($f_2(x) = P_G(x, <N_b> + <N_s>, <N_b> + N_s>)$). The probability κ that the "standard physics" (curve I ($f_1(x) = P_G(x, <N_b>, <N_b>)$)) can imitate new physics (i.e. the probability that we measure "new physics" but we think that it is described by the "standard physics") is described by common area of curve I and II.

7cm!sbfig1.eps

Figure 1: The probability density functions $f_{1,2}(x) \equiv P_G(x, \mu_{1,2}, \sigma^2)$ for $\mu_1 = <N_b> = 53$ and $\mu_2 = <N_b> + <N_s> = 104$.

Numerically, we find that

[1] With a precision defined by the tails (see Section 5).

$$\begin{aligned}
\kappa &= \frac{1}{\sqrt{2\pi}\sigma_2}\int_{-\infty}^{\sigma_1\sigma_2} exp[-\frac{(x-\sigma_2^2)^2}{2\sigma_2^2}]dx + \frac{1}{\sqrt{2\pi}\sigma_1}\int_{\sigma_1\sigma_2}^{\infty} exp[-\frac{(x-\sigma_1^2)^2}{2\sigma_1^2}]dx \\
&= \frac{1}{\sqrt{2\pi}}[\int_{-\infty}^{\sigma_1-\sigma_2} exp[-\frac{y^2}{2}]dy + \int_{\sigma_2-\sigma_1}^{\infty} exp[-\frac{y^2}{2}]dy] \qquad (4) \\
&= 1 - erf(\frac{\sigma_2-\sigma_1}{\sqrt{2}}).
\end{aligned}$$

Here $\sigma_1 = \sqrt{N_b}$ and $\sigma_2 = \sqrt{N_b + N_s}$. The transformation of the distributions to standard normal distribution and the exploitation of the equality

$$\frac{x_0 - \sigma_1^2}{\sigma_1} = -\frac{x_0 - \sigma_2^2}{\sigma_2}$$

allows one to find the point x_0 of the intersection of the curves I and II.

Let us discuss the meaning of our definition (4). For $x \leq x_0 = \sigma_1\sigma_2$ we have $f_1(x) \geq f_2(x)$, i.e. the probability density of the standard physics realization is higher than the probability density of new physics realization. Therefore for $x \leq x_0$ we do not have any indication in favour of new physics. The probability that the number of events is less than x_0 is $\alpha = \displaystyle\int_{-\infty}^{x_0} f_2(x)dx$. For $x > x_0$ $f_2(x) > f_1(x)$ that gives evidence in favour of new physics existence. However the probability of the background events with $x > x_0$ is different from zero and is equal to $\beta = \displaystyle\int_{x_0}^{\infty} f_1(x)dx$. So we have two types of the errors. For $x \leq x_0$ we do not have any evidence in favour of new physics (even in this case the probabilty of new physics realization is different from zero). For $x > x_0$ we have evidence in favour of new physics. However for $x > x_0$ the fluctuations of the background can imitate new physics. So the probability that standard physics can imitate new physics has two components α and β and it is equal to $\kappa = \alpha + \beta$. If κ equals to 1 new physics will never be found in the experiment, if κ equals to 0 the first measurement with probability one has to answer the question about presence or absence of new physics (this case is not realized for Poisson distribution). In other words one can say that the area of intersection of the probability density functions of the pure background and the background plus signal is the measure of the future experiment undiscovery potential.

As follows from formula (4) the role of the significance S plays

$$S_{12} = \sigma_2 - \sigma_1 = \sqrt{N_b + N_s} - \sqrt{N_b}. \qquad (5)$$

Note that in refs.[7] the following criterion of the signal discovery has been used. The signal was assumed to be observable if $(1-\epsilon)\cdot 100\%$ upper confidence level for the background event rate is equal to $(1-\epsilon)\cdot 100\%$ lower confidence level for background plus signal ($\epsilon = 0.01-0.05$). The corresponding significance is similar to our significance S_{12}. The difference is that in our approach the probability κ that new physics is described by standard physics is equal to 2ϵ.

It means that for $S_{12} = 1,2,3,4,5,6$ the probability κ is correspondingly $\kappa = 0.31, 0.046, 0.0027, 6.3\cdot 10^{-5},\ 5.7\cdot\ 10^{-7}, 2.0\cdot\ 10^{-9}$ in accordance with a general picture. As it has been mentioned in the Introduction two definitions of the significance are mainly used in the literature: $S_1 = \dfrac{<N_s>}{\sqrt{<N_b>}}$ [4] and $S_2 = \dfrac{<N_s>}{\sqrt{<N_s>+<N_b>}}$ [5]. The significance S_{12} is expressed in terms of the significances S_1 and S_2 as $S_{12} = \dfrac{S_1 S_2}{S_1+S_2}$.

For $<N_b> \ggg <N_s>$ (the search for Higgs boson through $H\to\gamma\gamma$ decay mode) we find that

$$S_{12} \approx 0.5\ S_1 \approx 0.5\ S_2. \tag{6}$$

It means that for $S_1 = 5$ (according to a common convention the 5σ confidence level means a new physics discovery) the real significance is $S_{12} = 2.5$, that corresponds to $\kappa = 1.24\%$ (Fig.2).

7cm!sbfig2.eps

Figure 2: The dependence of κ on number of signal events for cases $S_1 = 5,\ S_2 = 5$ and $S_{12} = 2.5$.

For the case $N_s = kN_b$, $S_{12} = k_{12}S_2$, where for $k = 0.5, 1, 4, 10$ the values of k_{12} are $k_{12} = 0.55, 0.59, 0.69, 0.77$, correspondingly. For not too high values of $<N_b>$ and $<N_b>+<N_s>$, we have to compare the Poisson distributions directly. Again for the Poisson distribution $P(n,<n>)$ with the area of definition for nonnegative integers we can define $P(x,<n>)$ for real x as

$$\tilde{P}(x,<n>) = \begin{cases} 0, & x<0, \\ P([x],<n>), & x\geq 0. \end{cases} \tag{7}$$

It is evident that

$$\int_{-\infty}^{\infty} \tilde{P}(x, < n >)dx = 1. \tag{8}$$

So, the generalization of the previous determination of κ in our case is straightforward, namely, κ is nothing but the common area of the curves described by $\tilde{P}(x, < N_b >)$ (curve I) and $\tilde{P}(x, < N_b > + < N_s >)$ (curve II) (see, Fig.3).

7cm!sbfig3.eps

Figure 3: The probability density functions $f_{1,2}(x) \equiv \tilde{P}(x, \mu_{1,2})$ for $\mu_1 = < N_b > = 1$ and $\mu_2 =< N_b > + < N_s >= 6$.

One can find that $\kappa = \alpha + \beta$, where

$$\alpha = \sum_{n=0}^{n_0} \frac{(< N_b > + < N_s >)^n}{n!} e^{-(<N_b>+<N_s>)} = 1 - F(2 < N_b > +2 < N_s > |2n_0 + 2),$$

$$\beta = \sum_{n=n_0+1}^{\infty} \frac{(< N_b >)^n}{n!} e^{-<N_b>} = F(2 < N_b > |2n_0 + 2),$$

$F(\chi^2|n) = \dfrac{1}{2^{\frac{n}{2}}\Gamma(\frac{n}{2})} \displaystyle\int_0^{\chi^2} e^{-\frac{t}{2}} t^{\frac{n}{2}-1} dt$ (see, for example, [9]) and $n_0 = [\dfrac{< N_s >}{ln(1 + \frac{<N_s>}{<N_b>})}]$.

Numerical results are presented in Tables 1-6.

As it follows from these Tables for finite values of $< N_s >$ and $< N_b >$ the deviation from asymptotic formula (4) is essential. For instance, for $N_s = 5$, $N_b = 1$ $(S_1 = 5)$ $\kappa = 14.2\%$. For $N_s = N_b = 25$ $(S_1 = 5)$ $\kappa = 3.8\%$, whereas asymptotically for $N_s \gg 1$ we find $\kappa = 1.24\%$. Similar situation takes place for $N_s \sim N_b$.

3 An account of systematic errors related to nonexact knowledge of background and signal cross sections

In the previous section we determined the statistical error κ (the probability that "new physics" is described by "standard physics"). In this section we investigate the influence of the systematical errors related to a nonexact knowledge of the background and signal cross sections on the probability κ not to confuse a new physics with the old one.

Denote the Born background and signal cross sections as σ_b^0 and σ_s^0. An account of one loop corrections leads to $\sigma_b^0 \to \sigma_b^0(1+\delta_{1b})$ and $\sigma_s^0 \to \sigma_s^0(1+\delta_{1s})$, where typically δ_{1b} and δ_{1s} are O(0.5).

Two loop corrections at present are not known. So, we can assume that the uncertainty related with nonexact knowledge of cross sections is around δ_{1b} and δ_{1s} correspondingly. In other words, we assume that the exact cross sections lie in the intervals $(\sigma_b^0, \sigma_b^0(1+2\delta_{1b}))$ and $(\sigma_s^0, \sigma_s^0(1+2\delta_{1s}))$. The average number of background and signal events lie in the intervals

$$(< N_b^0 >, < N_b^0 > (1+2\delta_{1b})) \tag{9}$$

and

$$(< N_s^0 >, < N_s^0 > (1+2\delta_{1s})), \tag{10}$$

where $< N_b^0 >= \sigma_b^0 \cdot L$, $< N_s^0 >= \sigma_s^0 \cdot L$.

To determine the probability that the new physics is described by the old one, we again have to compare two Poisson distributions with and without new physics but in distinction from Section 2 we have to compare the Poisson distributions in which the average numbers lie in some intervals. So, a priori the only thing we know is that the average numbers of background and signal events lie in the intervals (9) and (10), but we do not know the exact values of $< N_b >$ and $< N_s >$. To determine the probability that the new physics is described by the old one, consider the worst case [2] when we think that new physics is described by the minimal number of average events

$$< N_b^{min} >=< N_b^0 > + < N_s^0 > . \tag{11}$$

Due to the fact that we do not know the exact value of the background cross section, consider the worst case when the average number of background events is equal to $< N_b^0 > (1+2\delta_{1b})$. So, we have to compare the Poisson distributions with $< n >=< N_b^0 > + < N_s^0 >=$

$< N_b^0 > (1+2\delta_{1b}) + (< N_s^0 > -2\delta_{1b} < N_b^0 >)$ and $< n >=< N_b^0 > (1+2\delta_{1b})$. Using the result of the previous Section, we find that for case $< N_b^0 > \gg 1, < N_s^0 > \gg 1$ the effective significance is

[2]There is a problem to determine systematic uncertainty probability distributions for theoretical predictions under consideration.

$$S_{12s} = \sqrt{< N_b^0 > + < N_s^0 >} - \sqrt{< N_b^0 > (1 + 2\delta_{1b})}. \qquad (12)$$

For the limiting case $\delta_{1b} \to 0$, we reproduce formula (5). For not too high values of $< N_b^0 >$ and $< N_s^0 >$, we have to use the results of the previous section (Tables 1-6).

As an example consider the case when $\delta_{1b} = 0.5$, $< N_s >= 100$, $< N_b >= 50$ (typical situation for sleptons search). In this case we find that

$$\begin{aligned}
S_1 &= \frac{< N_s >}{\sqrt{< N_b >}} = 14.1, \\
S_2 &= \frac{< N_s >}{\sqrt{< N_s > + < N_b >}} = 8.2 \\
S_{12} &= \sqrt{< N_b > + < N_s >} - \sqrt{< N_b >} = 5.2, \\
S_{12s} &= \sqrt{< N_b > + < N_s >} - \sqrt{2 < N_b >} = 2.25.
\end{aligned}$$

The difference between CMS adopted significance $S_2 = 8.2$ (that corresponds to the probability $\kappa = 0.24 \cdot 10^{-15}$) and the significance $S_{12s} = 2.25$ taking into account systematics related to nonexact knowledge of background cross section is factor 3.6. The direct comparison of the Poisson distributions with $< N_b > (1 + 2\delta_{1b}) = 100$ and $< N_b > (1 + 2\delta_{1b}) + < N_{s,eff} >$ ($< N_{s,eff} >=< N_s > -2\delta_{1b} < N_b >= 50$) gives $\kappa_s = 0.0245$.

Another example is with $< N_s >= 28$, $< N_b >= 8$ and $\delta_{1b} = 0.5$. For such example we have $S_1 = 9.9$, $S_2 = 4.7$, $S_{12} = 3.2$, $S_{12s} = 2.0$, $\kappa_s = 0.045$.

So, we see that an account of the systematics related to nonexact knowledge of background cross sections is very essential and it decreases the LHC SUSY discovery potential.

4 Estimation of exclusion limits on new physics

In this section we generalize the results of the previous sections to obtain exclusion limits on signal cross section (new physics).

Suppose we know the background cross section σ_b and we want to obtain bound on signal cross section σ_s which depends on some parameters (masses of new particles, coupling constants, ...) and describes some new physics beyond standard model. Again as in Section 2 we have to compare two Poisson distributions with and without new physics. The results of Section 2 are trivially generalized for the case of the estimation of exclusion limits on signal cross section and, hence, on parameters (masses, coupling constants, ...) of new physics.

Consider at first the case when $< N_b >= \sigma_b \cdot L \gg 1$, $< N_s >= \sigma_s \cdot L \gg 1$ and the Poisson distributions approach the Gaussian distributions. As it has been mentioned in Section 2 the common area of the Gaussian curves with background events and with background plus signal events is the probability that "new physics" can be described by the "standard physics". For instance, when we require the probability that "new physics" can be described by the "standard physics" is less or equal 10% (S_{12} in formula (5) is larger than 1.64) it means that the formula

$$\sqrt{< N_b > + < N_s >} - \sqrt{< N_b >} \leq 1.64 \tag{13}$$

gives us 90% exclusion limit on the average number of signal events $< N_s >$. In general case when we require the probability that "new physics" can be described by the "standard physics" is more or less ϵ the formula

$$\sqrt{< N_b > + < N_s >} - \sqrt{< N_b >} \leq S(\epsilon) \tag{14}$$

allows us to obtain $1 - \epsilon$ exclusion limit on signal cross section. Here $S(\epsilon)$ is determined by the formula (4) [3], i.e. we suppose that $\epsilon = \kappa$. It should be stressed that in fact the requirement that "new physics" with the probability more or equal to ϵ can be described by the "standard physics" is our definition of the exclusion limit as $(1 - \epsilon)$ probability for signal cross section. From the formula (14) we find that

$$\sigma_s \leq \frac{S^2(\epsilon)}{L} + 2S(\epsilon)\sqrt{\frac{\sigma_b}{L}}. \tag{15}$$

For the case of not large values of $< N_b >$ and $< N_s >$ we have to compare the Poisson distributions directly and the corresponding method has been formulated in Section 2. As an example in Table 7 we give 90% exclusion limits on the signal cross section for $L = 10^4 pb^{-1}$ and for different values of background cross sections.

Formulae (14), (15) do not take into account the influence of the systematical errors related to nonexact knowledge of the background cross sections on the exclusion limits for signal cross section. To take into account such systematics we have to use the results of Section 3. The corresponding generalization of the formulae (14) and (15) is straightforward, namely:

$$\sqrt{< N_b > + < N_s >} - \sqrt{< N_b > (1 + 2\delta_{1b})} \leq S(\epsilon), \tag{16}$$

$$\sigma_s \leq \frac{S^2(\epsilon)}{L} + 2S(\epsilon)\sqrt{\frac{\sigma_b(1 + 2\delta_{1b})}{L}} + 2\delta_{1b}\sigma_b. \tag{17}$$

[3] Note that $S(1\%) = 2.57$, $S(2\%) = 2.33$, $S(5\%) = 1.96$ and $S(10\%) = 1.64$

Remember that δ_{1b} describes theoretical uncertainty in the calculation of the background cross section. As an example, in Table 8 we give 90% exclusion limits on the signal cross section for $L = 10^4 pb^{-1}$, $2\delta_{1b} = 0.25$ and for different values of background cross sections.

Note that in refs.[9, 10] different and strictly speaking "ad hoc" methods to derive exclusion limits in future experiments has been suggested. As is seen from Fig.4 the essential differences in values of the exclusion limits take place. Let us compare these methods by the use of the equal probability test [11].

7cm!sbfig4.eps

Figure 4: Estimations of the 90% CL upper limit on the signal in a future experiment as a function of the expected background. The method proposed in ref. [10] gives the values of exclusion limit close to "Typical experiment" approach.

In order to estimate the various approaches of the exclusion limit determination we suppose that new physics exists, i.e. the value $< N_s >$ equals to one of the exclusion limits from Fig.4 and the value $< N_b >$ equals to the corresponding value of expected background. Then we apply the equal probability test to find critical value n_0 for hypotheses testing in future measurements. Here a zero hypothesis is the statement that new physics exists and an alternative hypothesis is the statement that new physics is absent. After calculation of the Type I error α (the probability that the number of observed events will be equal to or less than the critical value n_0) and the Type II error β (the probability that the number of observed events will be greater than the critical value n_0 in the case of absence of new physics) we can compare the methods. In Table 9 the comparison result is shown. As is seen from this Table the "Typical experiment" approach [10] gives too small values of exclusion limit. The difference in the 90% CL definition is the main reason of the difference between our result and the exclusion limit from ref. [9]. We require that $\epsilon = \kappa$. In ref [9] the criterion for determination exclusion limits: $\beta < \Delta$ and $\dfrac{\alpha}{1-\beta} < \epsilon$ is used, i.e. the experiment will observe with probability at least $1 - \Delta$ at most a number of events such that the limit obtained at the $1 - \epsilon$ confidence level excludes the corresponding signal [4].

[4]If we define ϵ as normalized κ ($\epsilon = \tilde{\kappa} = \dfrac{\kappa}{2-\kappa}$) we have the result close to ref. [9]. For example, $\kappa = 0.17$ corresponds to $\epsilon = 0.0929$, i.e. $1 - \epsilon \approx 0.9$.

5 The probability of new physics discovery

In section 2 we determined the probability κ that "new physics" can be described by the "standard physics". But it is also very important to determine the probabilty of new physics discovery in future experiment. According to common definition [1] the new physics discovery corresponds to the case when the probability that background can imitate signal is less than 5σ or in terms of the probabilty less than $5.7 \cdot 10^{-7}$ (here of course we neglect any possible systematical errors).

So we require that the probability $\beta(\Delta)$ of the background fluctuations for $n > n(\Delta)$ is less than Δ, namely

$$\beta(\Delta) = \sum_{n=n_0(\Delta)+1}^{\infty} P(< N_b >, n) \leq \Delta \tag{18}$$

The probability $1 - \alpha(\Delta)$ that the number of signal events will be bigger than $n_0(\Delta)$ is equal to

$$1 - \alpha(\Delta) = \sum_{n=n_0(\Delta)+1}^{\infty} P(< N_b > + < N_s >, n) \tag{19}$$

It should be stressed that Δ is a given number and $\alpha(\Delta)$ is a function of Δ. Usually physicists claim the discovery of phenomenon [1] if the probability of the background fluctuation is less than 5σ that corresponds to $\Delta_{dis} = 5.7 \cdot 10^{-7}$ [5]. So from the equation (18) we find $n_0(\Delta)$ and estimate the probabilty $1 - \alpha(\Delta)$ that an experiment will satisfy the discovery criterion.

As an example consider the search for standard Higgs boson with a mass $m_h = 110\ GeV$ at the CMS detector. For total luminosity $L = 3 \cdot 10^4 pb^{-1} (2 \cdot 10^4 pb^{-1})$ one can find [1] that $< N_b >= 2893(1929), < N_s >= 357(238)$, $S_1 = \dfrac{< N_s >}{\sqrt{< N_b >}} = 6.6(5.4)$. Using the formulae (18, 19) for $\Delta_{dis} = 5.7 \cdot 10^{-7}$ (5σ discovery criterion) we find that $1 - \alpha(\Delta_{dis}) = 0.96(0.73)$. It means that for total luminosity $L = 3 \cdot 10^4 pb^{-1} (2 \cdot 10^4 pb^{-1})$ the CMS experiment will discover at $\geq 5\sigma$ level standard Higgs boson with a mass $m_h = 110\ GeV$ with a probabilty 96(73) percent.

An account of uncertainties related to nonexact knowledge of background cross section is straightforward and it is based on the results of Section 3. Suppose uncertainty in the calculation of exact cross section is determined by parameter δ, i.e. the exact cross section lies in the interval $(\sigma_b, \sigma_b(1 + \delta))$ and the exact value

[5] The approximation of Poisson distribution by Gaussian for tails with area close to or less than Δ_{dis} is wrong.

of average number of events lies in the interval $(< N_b >, < N_b > (1+\delta))$. Taking into account formulae (18) and (19) we have the formulae

$$\beta(\Delta) = \sum_{n=n_0(\Delta)+1}^{\infty} P(< N_b > (1+\delta), n) \leq \Delta \qquad (20)$$

$$1 - \alpha(\Delta) = \sum_{n=n_0(\Delta)+1}^{\infty} P(< N_b > + < N_s >, n) \qquad (21)$$

As an application of formulae (20,21) consider the case $< N_s >=< N_b >= 100$ (typical case for the search for supersymmetry at LHC). For such values of $< N_s >$ and $< N_b >$ we have $S_1 = 10$, $S_2 = 7.1$, $S_{12} = 4.1$. For $\delta=$ 0., 0.1, 0.25, 0.5 we find that $1 - \alpha(\Delta_{dis}) = 0.9998$, 0.9938, 0.8793, 0.1696, correspondingly. So, we see that the uncertainty in the calculations of background cross section is extremely essential for the determination of the LHC discovery potential. Some other examples are presented in Tables 10-15.

Let us consider the random variable "luminosity of 5σ discovery claim" for predicted phenomenon in future experiment. Fig.5 illustrates the behaviour of this value for above example $< N_s >=< N_b >= 100$ at integrated luminosity $10^5 pb^{-1}$. As follow from Fig.5(b) we can point out average luminosity of 5σ discovery claim $\bar{L} = 0.3287 \cdot 10^5 pb^{-1}$ and estimate the accuracy of this prediction. As seems it is very important parameter for comparison of proposals of future experiments.

7cm!sbfig5.eps

Figure 5: The cumulative distribution function (a) and the behaviour of the probability distribution (b) of the random variable "luminosity of 5σ discovery claim" ($< N_s >=< N_b >= 100$ at integrated luminosity $10^5 pb^{-1}$).

6 Conclusions

In this paper we determined the probability to discover the new physics in the future experiments when the average number of background $< N_b >$ and signal events $< N_s >$ is known. We have found that in this case the role of significance plays $S_{12} = \sqrt{< N_b > + < N_s >} - \sqrt{< N_b >}$ in comparison with often used expressions for the significances $S_1 = \dfrac{< N_s >}{\sqrt{< N_b >}}$ and $S_2 = \dfrac{< N_s >}{\sqrt{< N_s > + < N_b >}}$.

For $< N_s > \ll < N_b >$ we have found that $S_{12} = 0.5 S_1 = 0.5 S_2$. For not too high values of $< N_s >$ and $< N_b >$, when the deviations from the Gaussian distributions are essential, our results are presented in Tables 1-6. We proposed a method for taking into account systematical errors related to the nonexact knowledge of background and signal events. An account of such kind of systematics is very essential in the search for supersymmetry and leads to an essential decrease in the probability to discover the new physics in the future experiments. We also proposed methods for the estimation of exclusion limits on new physics and the probability of the new physics discovery in future experiments.

We are indebted to M.Dittmar for very hot discussions and useful questions which were one of the motivations to perform this study. We are grateful to V.A.Matveev for the interest and useful comments. This work has been supported by RFFI grant 99-02-16956.

References

[1] *The Compact Muon Solenoid.* Technical Proposal, CERN/LHCC 94 -38, 1994.

[2] H.Cramér, *Mathematical Methods of Statistics,* Princeton Univ. Press, New Jersey, 1958

[3] Particle Data Group, Phys.Rev **D54** 1 (1996).

[4] See as an example:

V.Tisserand, *The Higgs to Two Photon Decay in the ATLAS Detector,* Talk given at the VI International Conference on Calorimetry in High Energy Physics, Frascati (Italy), June 8-14, 1996.

S.I.Bityukov and N.V.Krasnikov, *The Search for New Physics by the Measurement of the Four-jet Cross Section at LHC and TEVATRON,* Modern Physics Letters **A12**(1997)2011, also hep-ph/9705338.

M.Dittmar and H.Dreiner, *LHC Higgs Search with $l^+ \nu l^- \bar{\nu}$ final states,* CMS Note 97/083, October 1997.

[5] See as an example:

D.Denegri, L.Rurua and N.Stepanov, *Detection of Sleptons in CMS, Mass Reach,* CMS Note CMS TN/96-059, October 1996.

F.Charles, *Inclusive Search for Light Gravitino with the CMS Detector,* CMS Note 97/079, September 1997.

S.Abdullin, *Search for SUSY at LHC: Discovery and Inclusive Studies,* Presented at International Europhysics Conference on High Energy Physics, Jerusalem, Israel, August 19-26, 1997, CMS Conference Report 97/019, November 1997.

[6] S.I.Bityukov and N.V.Krasnikov, *The Search for Sleptons and Flavour Lepton Number Violation at LHC (CMS),* Preprint IFVE 97-67, Protvino, 1997, also hep-ph/9712358.

[7] N.Brown, *Degenerate Higgs and Z Boson at LEP200,* Z.Phys., **C49**, 1991, p.657.

H.Baer, M.Bisset, C.Kao and X.Tata, *Observability of $\gamma\gamma$ decays of Higgs bosons from supersymmetry at hadron supercolliders,* Phys.Rev., **D46**, 1992, p.1067.

[8] S.I.Bityukov and N.V.Krasnikov, *Towards the observation of signal over background in future experiments,* Preprint INR 0945a/98, Moscow, 1998, also S.I.Bityukov and N.V.Krasnikov, *New physics discovery potential in future experiments,* Modern Physics Letters **A13**(1998)3235.

[9] J.J.Hernandez, S.Navas and P.Rebecchi, *Estimating exclusion limits in prospective studies of searches,* Nucl.Instr.&Meth. A **378**, 1996, p.301, also J.J.Hernandez and S.Navas, *JASP: a program to estimate discovery and exclusion limits in prospective studies of searches,* Comp.Phys.Comm. **100**, 1997 p.119.

[10] T.Tabarelli de Fatis and A.Tonazzo, *Expectation values of exclusion limits in future experiments* (Comment), Nucl.Instr.&Meth. A **403**, 1998, p.151.

[11] S.I.Bityukov and N.V.Krasnikov, *On observability of signal over background,* Preprint IFVE 98-48, Protvino, 1998, also physics/9809037.

Table 1: The dependence of κ on $< N_s >$ and $< N_b >$ for $S_1 = 5$

$< N_s >$	$< N_b >$	κ
5	1	0.1423
10	4	0.0828
15	9	0.0564
20	16	0.0448
25	25	0.0383
30	36	0.0333
35	49	0.0303
40	64	0.0278
45	81	0.0260
50	100	0.0245
55	121	0.0234
60	144	0.0224
65	169	0.0216
70	196	0.0209
75	225	0.0203
80	256	0.0198
85	289	0.0193
90	324	0.0189
95	361	0.0185
100	400	0.0182
150	900	0.0162
500	10^4	0.0135
5000	10^6	0.0125

Table 2: The dependence of κ on $< N_s >$ and $< N_b >$ for $S_2 \approx 5$.

$< N_s >$	$< N_b >$	κ
26	1	$0.15 \cdot 10^{-4}$
29	4	$0.14 \cdot 10^{-3}$
33	9	$0.44 \cdot 10^{-3}$
37	16	$0.99 \cdot 10^{-3}$
41	25	$0.17 \cdot 10^{-2}$
45	36	$0.26 \cdot 10^{-2}$
50	49	$0.31 \cdot 10^{-2}$
55	64	$0.36 \cdot 10^{-2}$
100	300	$0.74 \cdot 10^{-2}$
150	750	$0.89 \cdot 10^{-2}$

Table 3: $< N_s >= \frac{1}{5} \cdot < N_b >$. The dependence of κ on $< N_s >$ and $< N_b >$.

$< N_s >$	$< N_b >$	κ
50	250	0.131
100	500	0.033
150	750	$0.89 \cdot 10^{-2}$
200	1000	$0.25 \cdot 10^{-2}$
250	1250	$0.74 \cdot 10^{-3}$
300	1500	$0.22 \cdot 10^{-3}$
350	1750	$0.65 \cdot 10^{-4}$
400	2000	$0.20 \cdot 10^{-4}$

Table 4: $< N_s >= \frac{1}{10} \cdot < N_b >$. The dependence of κ on $< N_s >$ and $< N_b >$.

$< N_s >$	$< N_b >$	κ
50	500	0.275
100	1000	0.123
150	1500	0.059
200	2000	0.029
250	2500	0.015
300	3000	$0.75 \cdot 10^{-2}$
350	3500	$0.38 \cdot 10^{-2}$
400	4000	$0.20 \cdot 10^{-2}$
450	4500	$0.11 \cdot 10^{-2}$
500	5000	$0.56 \cdot 10^{-3}$

Table 5: $< N_s > = < N_b >$. The dependence of κ on $< N_s >$ and $< N_b >$.

$< N_s >$	$< N_b >$	κ
2.	2.	0.561
4.	4.	0.406
6.	6.	0.308
8.	8.	0.239
10.	10.	0.188
12.	12.	0.150
14.	14.	0.121
16.	16.	0.098
18.	18.	0.079
20.	20.	0.064
24.	24.	0.042
28.	28.	0.028
32.	32.	0.019
36.	36.	0.013
40.	40.	$0.87 \cdot 10^{-2}$
50.	50.	$0.34 \cdot 10^{-2}$
60.	60.	$0.13 \cdot 10^{-2}$
70.	70.	$0.52 \cdot 10^{-3}$
80.	80.	$0.21 \cdot 10^{-3}$
100.	100.	$0.33 \cdot 10^{-4}$

Table 6: $< N_s >= 2\cdot < N_b >$. The dependence of κ on $< N_s >$ and $< N_b >$.

$< N_s >$	$< N_b >$	κ
2.	1.	0.463
4.	2.	0.294
6.	3.	0.200
8.	4.	0.141
10.	5.	0.102
12.	6.	0.073
14.	7.	0.052
16.	8.	0.037
18.	9.	0.027
20.	10.	0.020
24.	12.	0.011
28.	14.	$0.59 \cdot 10^{-2}$
32.	16.	$0.33 \cdot 10^{-2}$
36.	18.	$0.18 \cdot 10^{-2}$
40.	20.	$0.10 \cdot 10^{-2}$
50.	25.	$0.23 \cdot 10^{-3}$
60.	30.	$0.56 \cdot 10^{-4}$

Table 7: 90% exclusion limits on signal cross section for $L = 10^4 pb^{-1}$ and for different background cross section (everything in pb). The third column gives exclusion limit according to formula (15).

σ_b	σ_s	σ_s (continuous limit)
10^3	1.041	1.038
10^2	0.329	0.328
10	0.104	0.104
1	0.033	0.033
0.1	0.011	0.011
0.01	0.0036	0.0035
0.001	0.0013	0.0013
0.0001	0.00060	0.00060

Table 8: 90% exclusion limits on signal cross section for $L = 10^4 pb^{-1}$, $2\delta_{1b} = 0.25$ and for different background cross section (everything in pb). The third column gives exclusion limit according to formula (17).

σ_b	σ_s	σ_s (continuous limit)
10^3	251.25	251.16
10^2	25.37	25.37
10	2.62	2.62
1	0.29	0.29
0.1	0.037	0.037
0.01	0.0064	0.0064
0.001	0.0017	0.0017
0.0001	0.00064	0.00066

Table 9: The comparison of the different approaches to determination of the exclusion limits. The α and the β are the Type I and the Type II errors under the equal probability test. The κ equals to the sum of α and β.

	this paper				ref. [9]				ref. [10]			
N_b	N_s	α	β	κ	N_s	α	β	κ	N_s	α	β	κ
1	6.02	0.08	0.02	0.10	4.45	0.09	0.08	0.17	3.30	0.20	0.08	0.28
2	7.25	0.05	0.05	0.10	5.50	0.13	0.05	0.18	3.90	0.16	0.14	0.30
3	8.32	0.07	0.03	0.10	6.40	0.09	0.08	0.18	4.40	0.14	0.18	0.32
4	9.20	0.05	0.05	0.10	7.25	0.13	0.05	0.18	4.80	0.23	0.11	0.34
5	10.06	0.07	0.03	0.10	7.90	0.10	0.07	0.17	5.20	0.20	0.13	0.34
6	10.67	0.06	0.04	0.10	8.41	0.09	0.08	0.18	5.50	0.19	0.15	0.34
7	11.37	0.05	0.05	0.10	9.00	0.08	0.10	0.18	5.90	0.17	0.17	0.34
8	12.02	0.07	0.03	0.10	9.70	0.10	0.06	0.17	6.10	0.17	0.18	0.35
9	12.51	0.06	0.04	0.10	10.16	0.09	0.07	0.17	6.40	0.16	0.20	0.36
10	13.04	0.05	0.05	0.10	10.50	0.09	0.08	0.17	6.70	0.22	0.14	0.36
11	13.62	0.04	0.06	0.10	10.80	0.08	0.09	0.18	6.90	0.21	0.15	0.36

Table 10: The dependence of $1 - \alpha(\Delta_{dis})$ on $< N_s >$ and $< N_b >$ for $S_1 = 5$ and different values of δ.

$< N_s >$	$< N_b >$	$\delta = 0.0$	$\delta = 0.1$	$\delta = 0.25$	$\delta = 0.5$
5	1	0.0839	0.0839	0.0426	0.0426
10	4	0.1728	0.1174	0.0765	0.0288
15	9	0.2323	0.1321	0.0678	0.0132
20	16	0.2737	0.1783	0.0609	0.0071
25	25	0.3041	0.1779	0.0424	0.0020
30	36	0.3273	0.1480	0.0315	0.0007
35	49	0.3456	0.1502	0.0192	0.0001
40	64	0.3973	0.1305	0.0125	0.00003
45	81	0.4064	0.1157	0.0068	0.000004
50	100	0.4140	0.1042	0.0040	
55	121	0.4205	0.0950	0.0019	
60	144	0.4261	0.0876	0.0010	
65	169	0.4309	0.0723	0.0004	
70	196	0.4352	0.0606	0.0002	
75	225	0.4389	0.0516	0.0001	
80	256	0.4638	0.0444	0.00003	
85	289	0.4657	0.0387	0.00001	
90	324	0.4674	0.0306	0.000005	
95	361	0.4689	0.0245	0.000002	
100	400	0.4703	0.0199		
150	900	0.5041	0.0015		

Table 11: The dependence of $1 - \alpha(\Delta_{dis})$ on $< N_s >$ and $< N_b >$ for $S_2 \approx 5$ and different values of δ.

$< N_s >$	$< N_b >$	$\delta = 0.$	$\delta = 0.1$	$\delta = 0.25$	$\delta = 0.5$
26	1	0.9999	0.9999	0.9998	0.9998
29	4	0.9983	0.9968	0.9940	0.9825
33	9	0.9909	0.9779	0.9524	0.8423
37	16	0.9725	0.9473	0.8491	0.5730
41	25	0.9418	0.8806	0.6606	0.2457
45	36	0.9016	0.7622	0.4705	0.0848
50	49	0.8774	0.7058	0.3208	0.0222
55	64	0.8752	0.6206	0.2161	0.0057
100	300	0.7155	0.1307	0.0002	
150	750	0.6599	0.0119		

Table 12: $< N_s > = \frac{1}{5} \cdot < N_b >$. The dependence of $1 - \alpha(\Delta_{dis})$ on $< N_s >$ and $< N_b >$ for different values of δ.

$< N_s >$	$< N_b >$	$\delta = 0.$	$\delta = 0.1$
50	250	0.0408	0.0004
100	500	0.3032	0.0030
150	750	0.6599	0.0119
200	1000	0.8905	0.0301
250	1250	0.9735	0.0629
300	1500	0.9947	0.1127
350	1750	0.9992	0.1767
400	2000	0.9999	0.2595

Table 13: $< N_s > = \frac{1}{10} \cdot < N_b >$. The dependence of $1 - \alpha(\Delta_{dis})$ on $< N_s >$ and $< N_b >$.

$< N_s >$	$< N_b >$	$\delta = 0.$
50	500	0.0043
100	1000	0.0424
150	1500	0.1478
200	2000	0.3223
250	2500	0.5177
300	3000	0.6955
350	3500	0.8270
400	4000	0.9093

Table 14: $< N_s > = < N_b >$. The dependence of $1 - \alpha(\Delta_{dis})$ on $< N_s >$ and $< N_b >$ for different values of δ.

$< N_s >$	$< N_b >$	$\delta = 0.$	$\delta = 0.1$	$\delta = 0.25$	$\delta = 0.5$
2	2	0.0003	0.0003	0.0001	0.000005
4	4	0.0016	0.0007	0.0003	0.00003
6	6	0.0061	0.0030	0.0007	0.00006
8	8	0.0131	0.0041	0.0011	0.0001
10	10	0.0218	0.0081	0.0027	0.0002
12	12	0.0467	0.0206	0.0050	0.0003
14	14	0.0589	0.0283	0.0080	0.0004
16	16	0.0956	0.0512	0.0116	0.0007
18	18	0.1401	0.0609	0.0156	0.0007
20	20	0.1903	0.0925	0.0200	0.0012
24	24	0.3005	0.1402	0.0395	0.0017
28	28	0.4122	0.2280	0.0656	0.0031
32	32	0.5166	0.2821	0.0969	0.0050
36	36	0.6089	0.3773	0.1323	0.0073
40	40	0.7268	0.4703	0.1704	0.0101
50	50	0.8762	0.6688	0.3216	0.0181
60	60	0.9572	0.8309	0.4397	0.0332
70	70	0.9865	0.9206	0.5784	0.0612
80	80	0.9960	0.9648	0.7205	0.0850
100	100	0.9998	0.9938	0.8793	0.1696

Table 15: $< N_s > \;=\; 0.5\cdot < N_b >$. The dependence of $1 - \alpha(\Delta_{dis})$ on $< N_s >$ and $< N_b >$ for different values of δ.

$< N_s >$	$< N_b >$	$\delta = 0.$	$\delta = 0.1$	$\delta = 0.25$
2	4	0.0001	0.00002	0.000005
4	8	0.0003	0.0001	0.000009
6	12	0.0010	0.0002	0.00003
8	16	0.0017	0.0005	0.00004
10	20	0.0040	0.0009	0.00005
12	24	0.0071	0.0012	0.0001
14	28	0.0111	0.0023	0.0001
16	32	0.0156	0.0025	0.0002
18	36	0.0207	0.0039	0.0002
20	40	0.0341	0.0056	0.0003
24	48	0.0589	0.0099	0.0005
28	56	0.0886	0.0192	0.0008
32	64	0.1424	0.0259	0.0011
36	72	0.1796	0.0402	0.0013
40	80	0.2442	0.0575	0.0021
50	100	0.4140	0.1042	0.0040
60	120	0.5692	0.1947	0.0074
70	140	0.7187	0.2762	0.0118
80	160	0.8250	0.3820	0.0195
100	200	0.9456	0.5765	0.0408